TIME SERIES CONTINUOUS MODELING FOR IMPUTATION AND FORECASTING WITH IMPLICIT NEURAL REPRESENTATIONS

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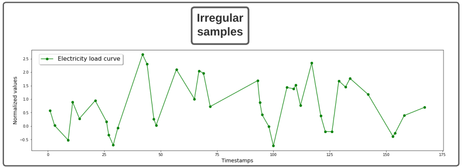
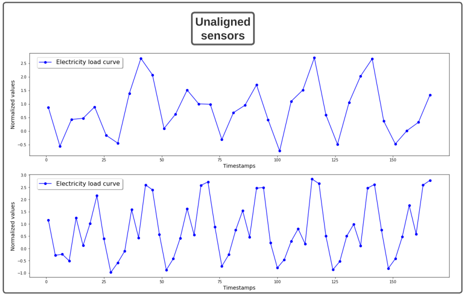
Time Series = continuous phenomena / observe partially

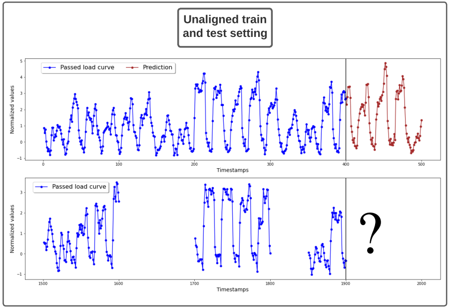
- Modeling Time Series as a continuous function

$\Rightarrow$ Deal with irregular sampling / unaligned sensors

$\Rightarrow$ Unified framework for Data imputation + Forecasting

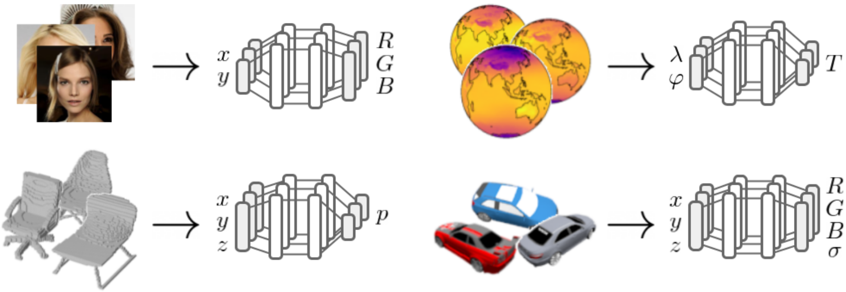






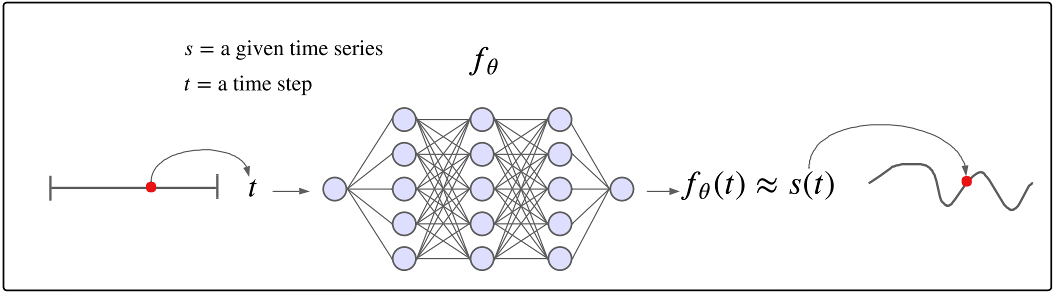
Technical options

- Gaussian Processes [Williams and Rasmussen, 2006]
- Neural Processes [Kim et al., 2019]
- Specific Architecture (e.g. mTAN) [Shukla and Marlin, 2021]
- Implicit Neural Representation (INR) [Dupont et al., 2022]



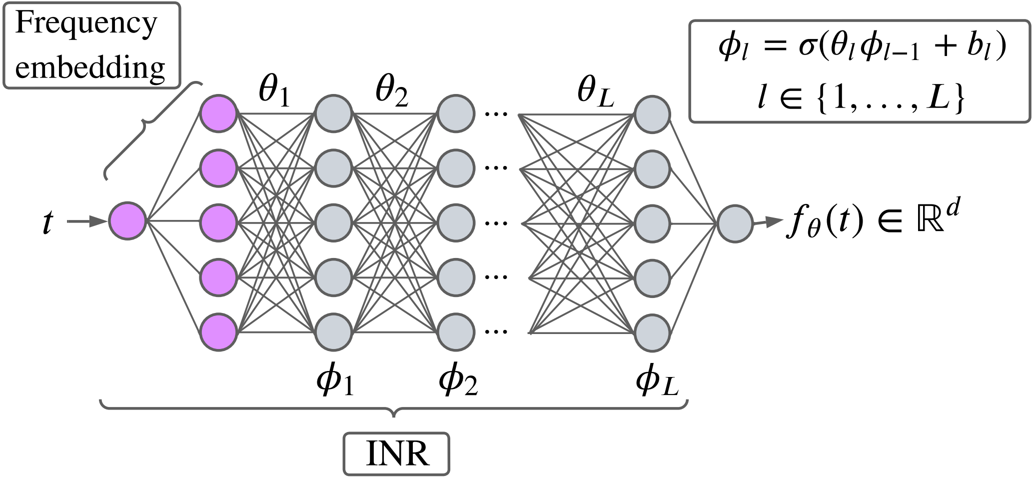
Implicit Neural Representation for Time Series

- A first attempt: DeepTime [Woo et al., 2022]
- Room for improvement:
  - Not designed for data imputation (forecasting only)
  - ≈ Ridge Regression on sampled Fourier descriptors

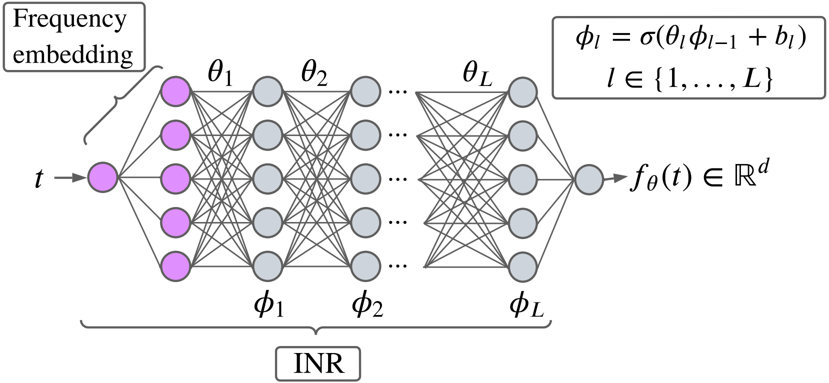


TimeFlow architecture

NeRF encoding illustration [Mildenhall et al., 2021]

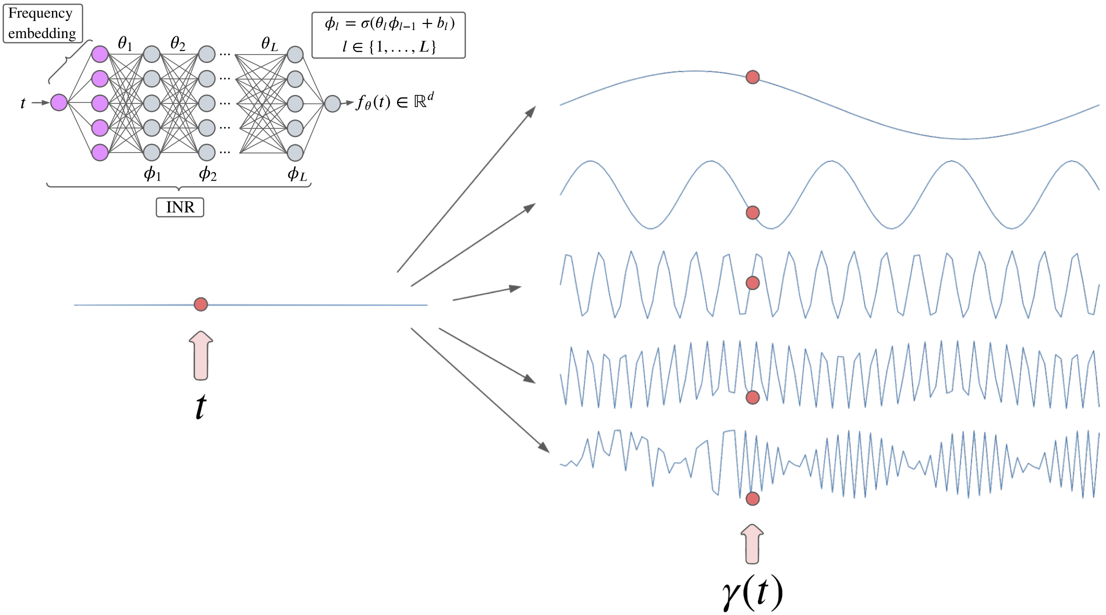


NeRF encoding illustration [Mildenhall et al., 2021]

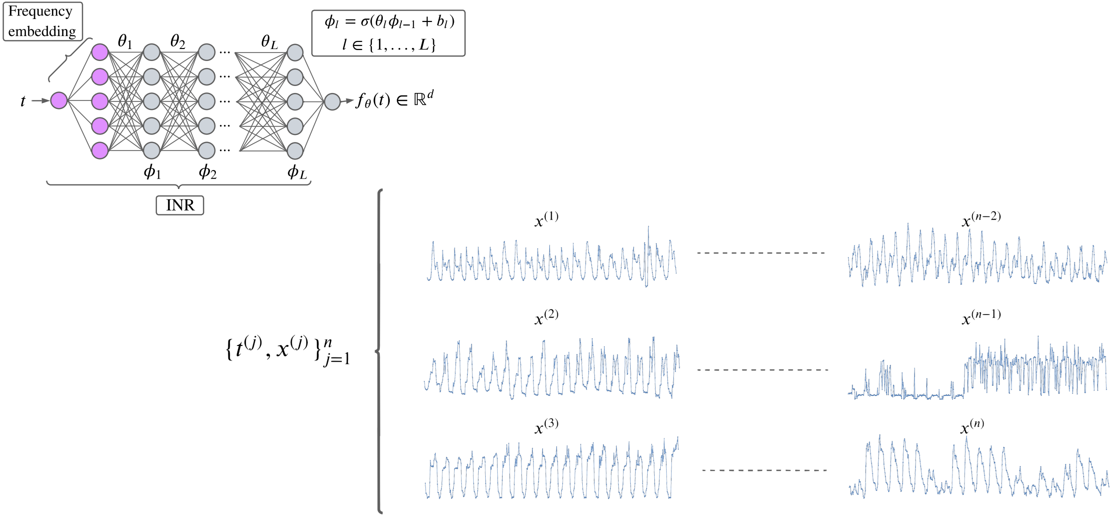


1. NeRF encoding : $t \rightarrow \gamma(t)$, N frequency bands
   $\gamma(t) := (\sin(\pi t), \cos(\pi t), \cdots, \sin(2^N \pi t), \cos(2^N \pi t))$
2. Then $\gamma(t) \rightarrow \text{MLP}(\gamma(t); \theta)$
   Activation functions are ReLU (i.e. $ReLU(x) = max(0, x)$)

NeRF encoding illustration [Mildenhall et al., 2021]

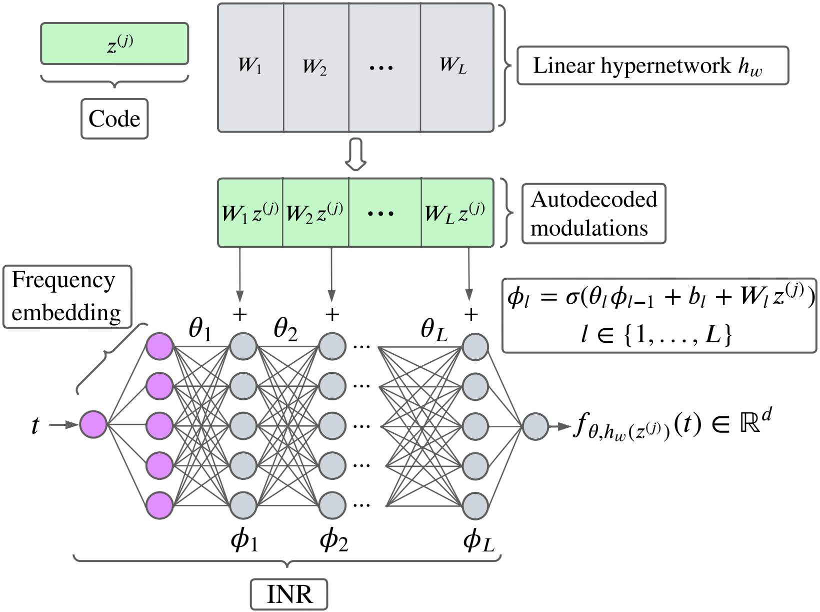


Nice to fit a sample, but how to deal with a dataset?

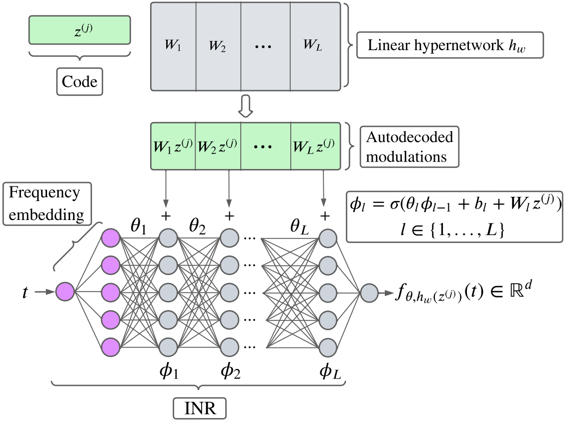


- Solution → **Hypernetwork that modulate the INR** [Dupont et al., 2022, Klocek et al., 2019, Sitzmann et al., 2020]

Hypernetwork and auto-decoding [Dupont et al., 2022, Yin et al., 2022]



Insight on θ, w and the $z^{(j)}$



- $\gamma(t)(= \phi_0) \in \mathbb{R}^{64}$, $z^{(j)} \in \mathbb{R}^{128}$
- $\phi_{\ell>0} \in \mathbb{R}^{256}$
- MLP: 5 layer

- $z^{(j)}$: instance coding
- θ and w = shared information across all samples
- MSE Loss

- **Training:** [Zintgraf et al., 2019] inner+outer loops

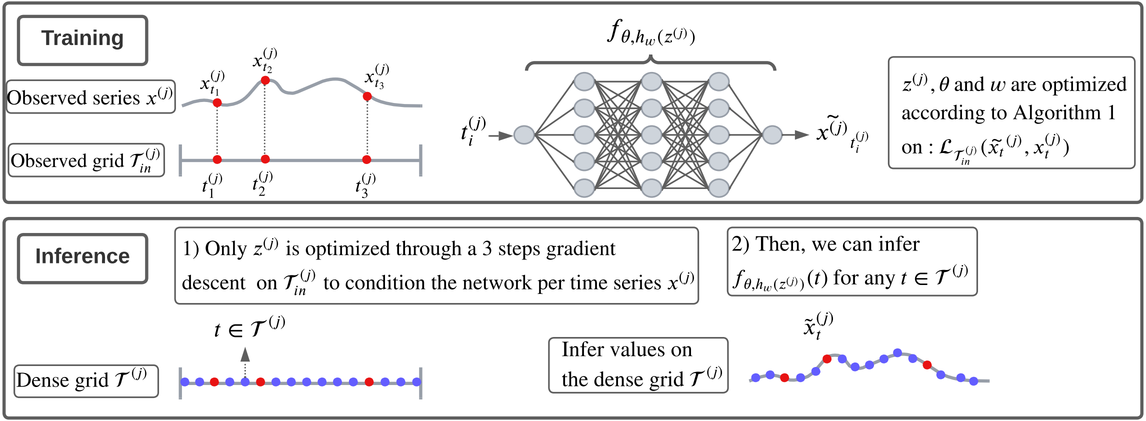
i) Sample adaptation = freeze (θ, w) + 3 grad. steps on $z^{(j)}$ [Second order grad. (Hessien comput.)]

o) (θ, w) optimization

- **Inference:** i) + forward not so fast...

Experiments

Imputation


Training
Observed series $x^{(j)}$
Observed grid $\mathcal{T}_{in}^{(j)}$
$x_{t_1}^{(j)}$ $x_{t_2}^{(j)}$ $x_{t_3}^{(j)}$
$t_1^{(j)}$ $t_2^{(j)}$ $t_3^{(j)}$
$f_{\theta,h_w(z^{(j)})}$
$t_i^{(j)}$
$\tilde{x}^{(j)}_{t_i^{(j)}}$
$z^{(j)}, \theta$ and w are optimized according to Algorithm 1 on : $\mathcal{L}_{\mathcal{T}_{in}^{(j)}}(\tilde{x}_t^{(j)}, x_t^{(j)})$
Inference
1) Only $z^{(j)}$ is optimized through a 3 steps gradient descent on $\mathcal{T}_{in}^{(j)}$ to condition the network per time series $x^{(j)}$
2) Then, we can infer $f_{\theta,h_w(z^{(j)})}(t)$ for any $t \in \mathcal{T}^{(j)}$
$t \in \mathcal{T}^{(j)}$
Dense grid $\mathcal{T}^{(j)}$
Infer values on the dense grid $\mathcal{T}^{(j)}$
$\tilde{x}_t^{(j)}$

We compare to a wide range of baselines on three datasets

Table 1: Mean MAE imputation results on the missing grid only. τ stands for the subsampling rate. Bold results are best, underlined results are second best.

		Continuous methods				Discrete methods			
	τ	TimeFlow	DeepTime	mTAN	Neural Process	CSDI	SAITS	BRITS	TIDER
Electricity	0.05	**0.324 ± 0.013**	0.379 ± 0.037	0.575 ± 0.039	0.357 ± 0.015	0.462 ± 0.021	0.384 ± 0.019	0.329 ± 0.015	0.427 ± 0.010
	0.10	**0.250 ± 0.010**	0.333 ± 0.034	0.412 ± 0.047	0.417 ± 0.057	0.398 ± 0.072	0.308 ± 0.011	0.287 ± 0.015	0.399 ± 0.009
	0.20	**0.225 ± 0.008**	0.244 ± 0.013	0.342 ± 0.014	0.320 ± 0.017	0.341 ± 0.068	0.261 ± 0.008	0.245 ± 0.011	0.391 ± 0.010
	0.30	**0.212 ± 0.007**	0.240 ± 0.014	0.335 ± 0.015	0.300 ± 0.022	0.277 ± 0.059	0.236 ± 0.008	0.221 ± 0.008	0.384 ± 0.009
	0.50	0.194 ± 0.007	0.227 ± 0.012	0.340 ± 0.022	0.297 ± 0.016	**0.168 ± 0.003**	0.209 ± 0.008	0.193 ± 0.008	0.386 ± 0.009
Solar	0.05	**0.095 ± 0.015**	0.190 ± 0.020	0.241 ± 0.102	0.115 ± 0.015	0.374 ± 0.033	0.142 ± 0.016	0.165 ± 0.014	0.291 ± 0.009
	0.10	**0.083 ± 0.015**	0.159 ± 0.013	0.251 ± 0.081	0.114 ± 0.014	0.375 ± 0.038	0.124 ± 0.018	0.132 ± 0.015	0.276 ± 0.010
	0.20	**0.072 ± 0.015**	0.149 ± 0.020	0.314 ± 0.035	0.109 ± 0.016	0.217 ± 0.023	0.108 ± 0.014	0.109 ± 0.012	0.270 ± 0.010
	0.30	**0.061 ± 0.012**	0.135 ± 0.014	0.338 ± 0.05	0.108 ± 0.016	0.156 ± 0.002	0.100 ± 0.015	0.098 ± 0.012	0.266 ± 0.010
	0.50	**0.054 ± 0.013**	0.098 ± 0.013	0.315 ± 0.080	0.107 ± 0.015	0.079 ± 0.011	0.094 ± 0.013	0.088 ± 0.013	0.262 ± 0.009
Traffic	0.05	0.283 ± 0.016	**0.246 ± 0.010**	0.406 ± 0.074	0.318 ± 0.014	0.337 ± 0.045	0.293 ± 0.007	0.261 ± 0.010	0.363 ± 0.007
	0.10	**0.211 ± 0.012**	0.214 ± 0.007	0.319 ± 0.025	0.288 ± 0.018	0.288 ± 0.017	0.237 ± 0.006	0.245 ± 0.009	0.362 ± 0.006
	0.20	**0.168 ± 0.006**	0.216 ± 0.006	0.270 ± 0.012	0.271 ± 0.011	0.269 ± 0.017	0.197 ± 0.005	0.224 ± 0.008	0.361 ± 0.006
	0.30	**0.151 ± 0.007**	0.172 ± 0.008	0.251 ± 0.006	0.259 ± 0.012	0.240 ± 0.037	0.180 ± 0.006	0.197 ± 0.007	0.355 ± 0.006
	0.50	**0.139 ± 0.007**	0.171 ± 0.005	0.278 ± 0.040	0.240 ± 0.021	0.144 ± 0.022	0.160 ± 0.008	0.161 ± 0.060	0.354 ± 0.007
TimeFlow improvement		/	24.14 %	50.53 %	31.61 %	36.12 %	20.33 %	18.90 %	53.40 %

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	TimeFlow	DeepTime	NeuralProcess	mTAN	SAITS	BRITS	TIDER
Number of parameters	602k	1315k	248k	113k	11 137k	6 220k	1 034k

Figure 1: Number of parameters for each DL methods on the imputation task on the Electricity dataset.

Qualitative comparison with BRITS

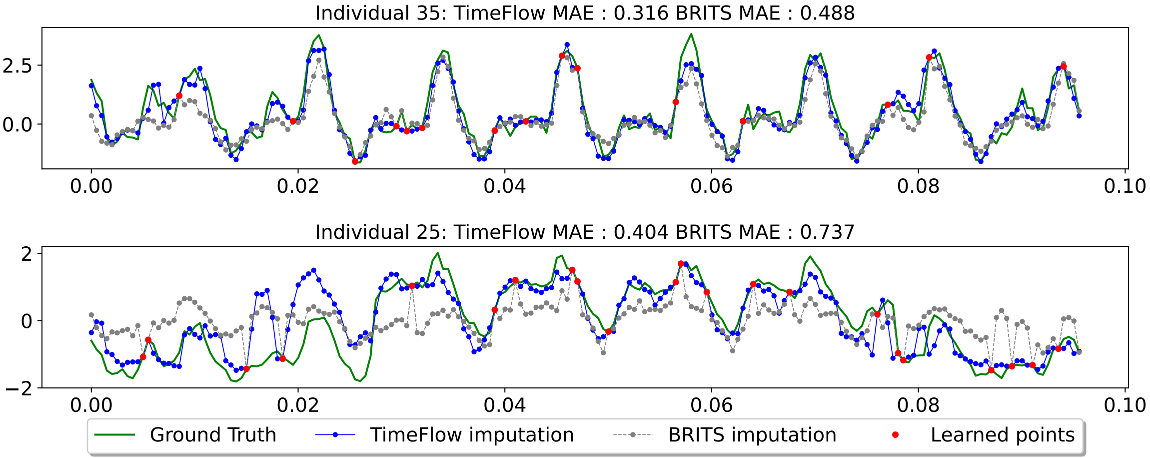
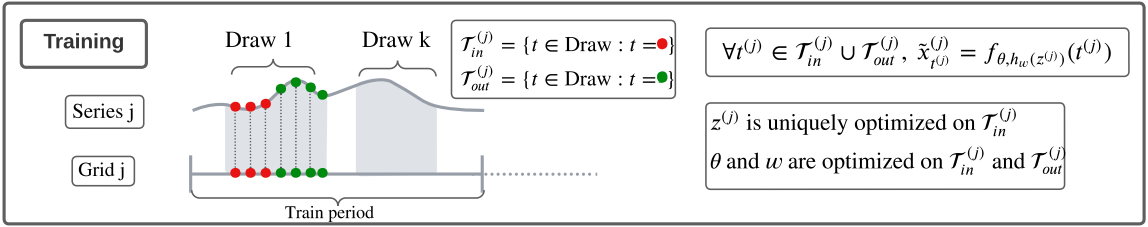
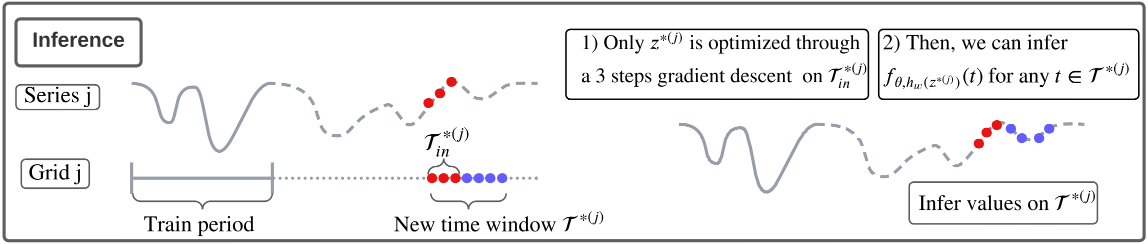


Figure 2: *Electricity dataset*. TimeFlow imputation (blue line) and BRITS imputation (gray line) with 10% of known point (red points) on the eight first days of samples 35 (top) and 25 (bottom).

Forecasting





Wide range of baselines on three datasets

Table 2: Mean MAE forecast results for adjacent time windows. H stands for the horizon. Bold results are best, underline results are second best.

		Continuous methods			Discrete methods			
	H	TimeFlow	DeepTime	Neural Process	Patch-TST	DLinear	AutoFormer	Informer
Electricity	96	<u>0.218 ± 0.017</u>	0.240 ± 0.027	0.392 ± 0.045	**0.214** ± **0.020**	0.236 ± 0.035	0.310 ± 0.031	0.293 ± 0.0184
	192	<u>0.238 ± 0.012</u>	0.251 ± 0.023	0.401 ± 0.046	**0.225** ± **0.017**	0.248 ± 0.032	0.322 ± 0.046	0.336 ± 0.032
	336	<u>0.265 ± 0.036</u>	0.290 ± 0.034	0.434 ± 0.075	**0.242** ± **0.024**	0.284 ± 0.043	0.330 ± 0.019	0.405 ± 0.044
	720	<u>0.318 ± 0.073</u>	0.356 ± 0.060	0.605 ± 0.149	**0.291** ± **0.040**	0.370 ± 0.086	0.456 ± 0.052	0.489 ± 0.072
SolarH	96	**0.172** ± **0.017**	<u>0.197 ± 0.002</u>	0.221 ± 0.048	0.232 ± 0.008	0.204 ± 0.002	0.261 ± 0.053	0.273 ± 0.023
	192	**0.198** ± **0.010**	<u>0.202 ± 0.014</u>	0.244 ± 0.048	0.231 ± 0.027	0.211 ± 0.012	0.312 ± 0.085	0.256 ± 0.026
	336	<u>0.207 ± 0.019</u>	**0.200** ± **0.012**	0.241 ± 0.005	0.254 ± 0.048	0.212 ± 0.019	0.341 ± 0.107	0.287 ± 0.006
	720	**0.215** ± **0.016**	<u>0.240 ± 0.011</u>	0.403 ± 0.147	0.271 ± 0.036	0.246 ± 0.015	0.368 ± 0.006	0.341 ± 0.049
Traffic	96	<u>0.216 ± 0.033</u>	0.229 ± 0.032	0.283 ± 0.028	**0.201** ± **0.031**	0.225 ± 0.034	0.299 ± 0.080	0.324 ± 0.113
	192	<u>0.208 ± 0.021</u>	0.220 ± 0.020	0.292 ± 0.023	**0.195** ± **0.024**	0.215 ± 0.022	0.320 ± 0.036	0.321 ± 0.052
	336	<u>0.237 ± 0.040</u>	0.247 ± 0.033	0.305 ± 0.039	**0.220** ± **0.036**	0.244 ± 0.035	0.450 ± 0.127	0.394 ± 0.066
	720	**0.266** ± **0.048**	0.290 ± 0.045	0.339 ± 0.037	<u>0.268 ± 0.050</u>	0.290 ± 0.047	0.630 ± 0.043	0.441 ± 0.055
TimeFlow improvement		/	6.56 %	30.79 %	2.64 %	7.30 %	35.43 %	33.07 %

Forecast on sparsely observed look-back window (1/2)

Table 3: MAE results for forecasting with missing values in the look-back window. τ stands for the percentage of observed values in the look-back window. Best results are in bold.

			TimeFlow		DeepTime		Neural Process	
	H	τ	Imputation error	Forecast error	Imputation error	Forecast error	Imputation error	Forecast error
Electricity	96	0.5	**0.151** $\pm$ **0.003**	**0.239** $\pm$ **0.013**	0.209 $\pm$ 0.004	0.270 $\pm$ 0.019	0.460 $\pm$ 0.048	0.486 $\pm$ 0.078
		0.2	**0.208** $\pm$ **0.006**	**0.260** $\pm$ **0.015**	0.249 $\pm$ 0.006	0.296 $\pm$ 0.023	0.644 $\pm$ 0.079	0.650 $\pm$ 0.095
		0.1	**0.272** $\pm$ **0.006**	**0.295** $\pm$ **0.016**	0.284 $\pm$ 0.007	0.324 $\pm$ 0.026	0.740 $\pm$ 0.083	0.737 $\pm$ 0.106
	192	0.5	**0.149** $\pm$ **0.004**	**0.235** $\pm$ **0.011**	0.204 $\pm$ 0.004	0.265 $\pm$ 0.018	0.461 $\pm$ 0.045	0.498 $\pm$ 0.070
		0.2	**0.209** $\pm$ **0.006**	**0.257** $\pm$ **0.013**	0.244 $\pm$ 0.007	0.290 $\pm$ 0.023	0.601 $\pm$ 0.075	0.626 $\pm$ 0.101
		0.1	**0.274** $\pm$ **0.010**	**0.289** $\pm$ **0.016**	0.282 $\pm$ 0.007	0.315 $\pm$ 0.025	0.461 $\pm$ 0.045	0.724 $\pm$ 0.090
Traffic	96	0.5	**0.180** $\pm$ **0.016**	**0.219** $\pm$ **0.026**	0.272 $\pm$ 0.028	0.243 $\pm$ 0.030	0.436 $\pm$ 0.025	0.444 $\pm$ 0.047
		0.2	**0.239** $\pm$ **0.019**	**0.243** $\pm$ **0.027**	0.335 $\pm$ 0.026	0.293 $\pm$ 0.027	0.596 $\pm$ 0.049	0.597 $\pm$ 0.075
		0.1	**0.312** $\pm$ **0.020**	**0.290** $\pm$ **0.027**	0.385 $\pm$ 0.025	0.344 $\pm$ 0.027	0.734 $\pm$ 0.102	0.731 $\pm$ 0.132
	192	0.5	**0.176** $\pm$ **0.014**	**0.217** $\pm$ **0.017**	0.241 $\pm$ 0.027	0.234 $\pm$ 0.021	0.477 $\pm$ 0.042	0.476 $\pm$ 0.043
		0.2	**0.233** $\pm$ **0.017**	**0.236** $\pm$ **0.021**	0.286 $\pm$ 0.027	0.276 $\pm$ 0.020	0.685 $\pm$ 0.109	0.678 $\pm$ 0.108
		0.1	**0.304** $\pm$ **0.019**	**0.277** $\pm$ **0.021**	0.331 $\pm$ 0.025	0.324 $\pm$ 0.021	0.888 $\pm$ 0.178	0.877 $\pm$ 0.174
TimeFlow improvement			/	/	18.97 %	11.87 %	61.88 %	58.41 %

Forecast on sparsely observed look-back window (2/2)

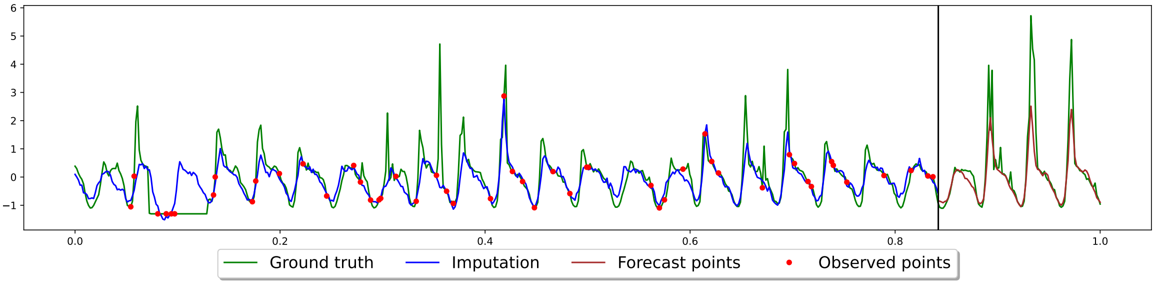
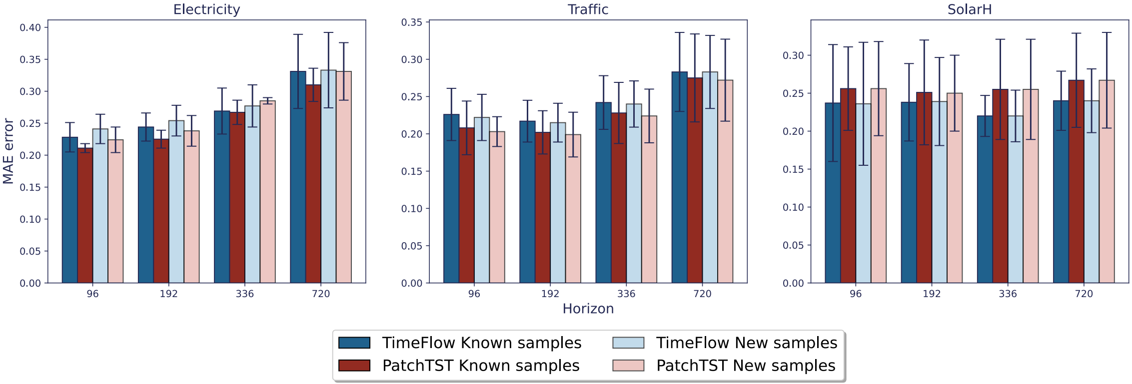


Figure 3: *Traffic dataset, sample 95*. In this figure, TimeFlow simultaneously imputes and forecasts at horizon 96 with a 10% partially observed look-back window of length 512.

Known *vs* New Samples

- TimeFlow *vs* PatchTST

$\Rightarrow$ Very close performances: Known $\approx$ New / TimeFlow $\approx$ PatchTST



Quantify uncertainty with TimeFlow ($\mathcal{L}$ is the pinball loss)

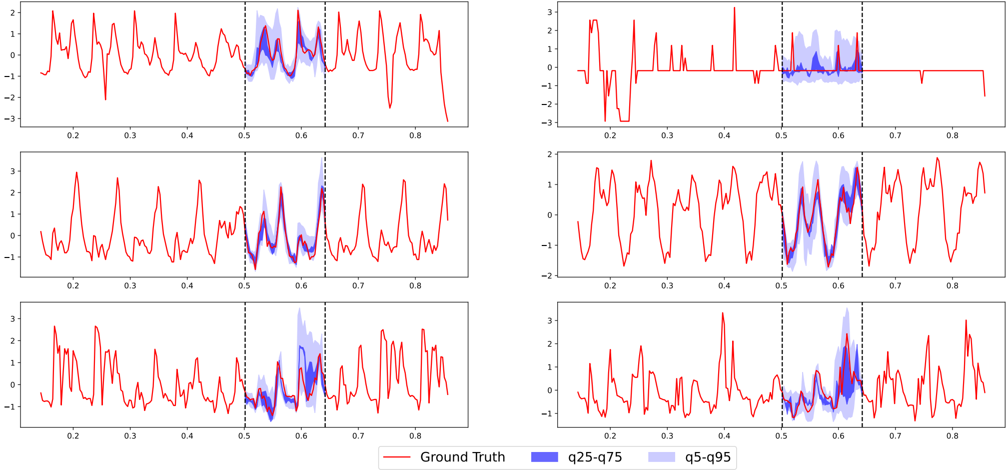


Figure 4: Quantifying uncertainty in block imputation of two missing days in the *Electricity* dataset.

Conclusion

Key takeaways

TimeFlow offers:

- Unified + continuous approach for time series imputation & forecasting.
- Adaptability to new contexts through meta-learning optimization.
- Very high performances in all situations
- Wide range of experiments to measure the benefits of all components

Limitation:

- Inference computation time (10-100 slower that competitors)

Perspectives:

- Moving to mutlivariate time-series

A team work

Time Series Continuous Modeling for Imputation and Forecasting with Implicit Neural Representations

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